Mock Exam 2

MATHEMATICS **9709**

Paper 3 Pure Mathematics 3

MARK SCHEME

Maximum Mark: 75

Published

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Marking scheme p3 mock 2021

Ans #01

Question	Answer	Marks
(a)	Draw two V-shaped graphs with one vertex on negative x-axis and one vertex on positive x-axis	M1
	Draw correct graphs related correctly to each other	A1
	State correct coordinates $-\frac{2}{3}a,\ 2a,\ \frac{4}{3}a,\ 4a$	A1
		3
(b)	Solve linear equation with signs of $3x$ different or solve non-modulus equation $(3x+2a)^2=(3x-4a)^2$	M1
	Obtain $x=\frac{1}{3}a$	A1
	Obtain $y=3a$	A1
		3

Question	Answer	Marks
(c)	State $x<\frac{1}{3}a$ (**FT** from part **(b)**)	B1FT
		1

Ans# 02

Question	Answer	Marks
(a)	Differentiate using the product rule to obtain $ax^2\cos 2x - bx^3\sin 2x$	M1
	Obtain $3x^2\cos 2x - 2x^3\sin 2x$	A1
	Equate first derivative to zero and confirm $x=\sqrt[3]{1.5x^2\cot 2x}$ AG	A1
		3
(b)	Consider sign of $x-\sqrt[3]{1.5x^2\cot 2x}$ or equivalent for 0.59 and 0.60	M1
	Obtain $-0.009...$ and $0.005...$ or equivalents and justify conclusion	A1
		2
(c)	Use iteration correctly at least once	M1
	Obtain final answer 0.596	A1
	Show sufficient iterations to 5 sf to justify answer or show sign change in interval [0.5955, 0.5965]	A1
		3

ANS #03

Question	Answer	Marks	Guidance
(i)	State $R=1.3$ or $\frac{10}{3}$	B1	Not $\sqrt{1.69}$
	Use appropriate trigonometry to find α	M1	AWRT ±1.18 rads, AWRT ±0.391 rads, AWRT $\pm67.4°$, AWRT $\pm22.6°$
	Obtain 67.38 with no errors seen	A1	AWRT
		3	
(ii)	Carry out correct method to find one value of θ between 0 and 360	M1	
	Obtain 240.6 (or 344.6)	A1	
	Carry out correct method to find second value of θ between 0 and 360	M1	Must be using either degrees throughout or radians throughout for M marks
	Obtain 344.6 (or 240.6)	A1	
	R = 2.6	4	
(iii)	Recognise expression as $[3-2R\cos(\theta+\alpha)]^2$ $(3-2.6)^2$	M1	
	Obtain $[3-2\times(-1.3)]^2$ and hence 31.36 or 31.4 $(3-(-2.6))^2$	A1	
	Obtain $[3-2\times1.3]^2$ and hence 0.16	A1	

Ans#4

	$x^3+px^2+30x+q=0$		
(a)	$1+5i$		B1
			(1)
(b)	$((x-(1+5i))(x-(1-5i)))=x^2-2x+26$ $((x-2)(x-(1\pm5i)))=x^2-(3\pm5i)x+2(1\pm5i)$	M1: 1. Attempt to expand or 2. Use sum and product of the complex roots. A1: Correct expression	M1A1
	$(x^2-2x+26)(x-2)=x^3+px^2+30x+q$	Uses their third factor with their quadratic with at least 4 terms in the expansion	M1
	$p=-4,\quad q=-52$	May be seen in cubic	A1, A1
OR	f(1+5i)=0 or f(1-5i)=0	Substitute one complex root to achieve 2 equations in p and / or q	M1
	$q-24p-44=0$ and $10p+40=0$	Both equations correct oe	A1
		Solving for p and q	M1
	$p=-4,\quad q=-52$	May be seen in cubic	A1, A1
			(5)

Ans# 4(b)

(b)(i)	Show a circle with centre 2 + 3i	**B1**
	Show a circle with radius 1 and centre not at the origin	**B1**
		2
b)(ii)	Carry out a complete method for finding the least value of arg z	**M1**
	Obtain answer $40.2°$ or 0.702 radians	**A1**

Ans#5

(a) $\frac{dx}{dt} = 6\cos t \times (-\sin t)$, $\frac{dy}{dt} = 2\cos 2t$ M1 A1

$\frac{dy}{dx} = \frac{2\cos 2t}{-6\cos t \sin t} = \frac{2\cos 2t}{-3\sin 2t} = -\frac{2}{3}\cot 2t$ M1 A1

(b) $-\frac{2}{3}\cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$ M1 A1

$\therefore\ (\frac{3}{2}, 1), (\frac{3}{2}, -1)$ A1

(c) $t = \frac{\pi}{6}$, $x = \frac{9}{4}$, $y = \frac{\sqrt{3}}{2}$, grad $= -\frac{2}{3\sqrt{3}}$ B1

$\therefore\ y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$ M1

$6\sqrt{3}\,y - 9 = -4x + 9$

$2x + 3\sqrt{3}\,y = 9$ A1

Question Number	Scheme		Marks
ANS #06	$l_1 : \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. Let $\theta =$ acute angle between l_1 and l_2.		
	Note: You can mark parts (a) and (b) together.		
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds μ and substitutes their μ into l_2	M1
	So, $\{\overrightarrow{OA}\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$	A1
			[2]
(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$	M1
	$\mathbf{k}:\ p - 3\lambda = -2 - 5\mu$ $p - 3(4) = -2 - 5(-1)$ $\underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3$ $p - 3(4) = 3$ $\underline{p = 15}$	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda =$ the **k** value of A found in part (a)", substitutes their λ and solves to give $p = ...$	M1
		$p = 15$	A1
			[3]
(c)	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	dM1 (A1 on ePEN)
	$\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}}$ $\theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1
			[3]
(d)	$\overrightarrow{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}$; $\overrightarrow{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{6^2 + 8^2 + (-10)^2}\ \{= 10\sqrt{2}\}$	**See notes**	M1
	$\dfrac{d}{10\sqrt{2}} = \sin\theta$	Writes down a correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe.	dM1
	$\{d = 10\sqrt{2}\sin 31.82...\}\ d = 7.456540753... = 7.46$ (3sf)	anything that rounds to 7.46	A1
			[3] **11**

(b)	**Alternative method for part (b)**		
	$\begin{Bmatrix} 3\times\mathbf{j}: & -9+3\lambda = 15+12\mu \\ \mathbf{k}: & p-3\lambda = -2+5\mu \end{Bmatrix}$ $p-9=13+7\mu$	Eliminates λ to write down an equation in p and μ	M1
	$p-9=13+7(-1)$ $\underline{p=15}$	Substitutes their μ and solves to give $p = ...$	M1
		$p=15$	A1
(d)	**Alternative Methods for part (d)** Let X be the foot of the perpendicular from B onto l_1		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\overrightarrow{OX} = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix}$		
	$\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}$		
	Method 1		
	$\overrightarrow{BX}\bullet\mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}\bullet\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -12+\lambda-66+9\lambda = 0$ leading to $10\lambda - 78 = 0 \Rightarrow \lambda = \frac{39}{5}$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{BX}\bullet\mathbf{d}_1 = 0$ and solves the resulting equation to find a value for λ.	M1
	$\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12+\frac{39}{5} \\ 22-3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$	Substitutes their value of λ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753...$	awrt 7.46	A1
	Method 2		
	Let $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9\lambda^2$ $= 10\lambda^2 - 156\lambda + 664$ So $\frac{d\beta}{d\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}$	Finds $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2$ in terms of λ, finds $\frac{d\beta}{d\lambda}$ and sets this result equal to 0 and finds a value for λ.	M1
	$\left\lvert\overrightarrow{BX}\right\rvert^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$	Substitutes their value of λ into their $\left\lvert\overrightarrow{BX}\right\rvert^2$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{\frac{278}{5}} = 7.456540753...$	awrt 7.46	A1

		Question 4 Notes
(a)	M1	Finds μ and substitutes their μ into l_2
	A1	Point of intersection of $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$. Allow $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$.
	Note	You cannot recover the answer for part (a) in part (c) or part (d).
(b)	M1	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$
	M1	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda =$ the **k** value of A" found in part (b).
	A1	$p = 15$
(c)	NOTE	**Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.**
	M1	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	Note	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	A1	anything that rounds to 31.82. This can also be achieved by 180 – 148.1796... = awrt 31.82
	Note	$\theta = 0.5553...^c$ is A0.
	Note	**M1A1 for** $\cos\theta = \left(\dfrac{0-16-60}{\sqrt{(0)^2+(4)^2+(-12)^2}.\sqrt{(-3)^2+(-4)^2+(5)^2}}\right) = \dfrac{-76}{\sqrt{160}.\sqrt{50}}$

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1\times\mathbf{d}_2 = \begin{pmatrix}0\\1\\-3\end{pmatrix}\times\begin{pmatrix}3\\4\\-5\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & -3\\ 3 & 4 & -5\end{vmatrix} = 7\mathbf{i}-9\mathbf{j}-3\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
$\sin\theta = \dfrac{\sqrt{(7)^2+(-9)^2+(3)^2}}{\sqrt{(0)^2+(1)^2+(-3)^2}.\sqrt{(3)^2+(4)^2+(-5)^2}}$	An attempt to apply the vector cross product formula	dM1 (A1 on ePEN)
$\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}}$ $\quad\theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1

(d)	M1	Full method for finding B **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$.
	dM1	**dependent on the first method mark being awarded.** Writes down correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where "their AB" is a value. and $\theta =$ "their θ" or stated as θ
	A1	anything that rounds to 7.46

ANS#7

(a) $u = \sin x \Rightarrow \dfrac{du}{dx} = \cos x$ B1

$I = \int \dfrac{6\cos x}{\cos^2 x(2-\sin x)}\, dx = \int \dfrac{6\cos x}{(1-\sin^2 x)(2-\sin x)}\, dx$ M1

$= \int \dfrac{6}{(1-u^2)(2-u)}\, du$ M1 A1

(b) $\dfrac{6}{(1+u)(1-u)(2-u)} \equiv \dfrac{A}{1+u} + \dfrac{B}{1-u} + \dfrac{C}{2-u}$

$6 \equiv A(1-u)(2-u) + B(1+u)(2-u) + C(1+u)(1-u)$ M1

$u = -1 \Rightarrow 6 = 6A \Rightarrow A = 1$ A1

$u = 1 \Rightarrow 6 = 2B \Rightarrow B = 3$ A1

$u = 2 \Rightarrow 6 = -3C \Rightarrow C = -2$ A1

$\therefore \dfrac{6}{(1-u^2)(2-u)} \equiv \dfrac{1}{1+u} + \dfrac{3}{1-u} - \dfrac{2}{2-u}$

(c) $x = 0 \Rightarrow u = 0,\ x = \frac{\pi}{6} \Rightarrow u = \frac{1}{2}$ M1

$I = \int_0^{\frac{1}{2}} \left(\dfrac{1}{1+u} + \dfrac{3}{1-u} - \dfrac{2}{2-u}\right) du$

$= [\ln|1+u| - 3\ln|1-u| + 2\ln|2-u|]_0^{\frac{1}{2}}$ M1 A2

$= (\ln\frac{3}{2} - 3\ln\frac{1}{2} + 2\ln\frac{3}{2}) - (0 + 0 + 2\ln 2)$ M1

$= 3\ln\frac{3}{2} + 3\ln 2 - 2\ln 2$

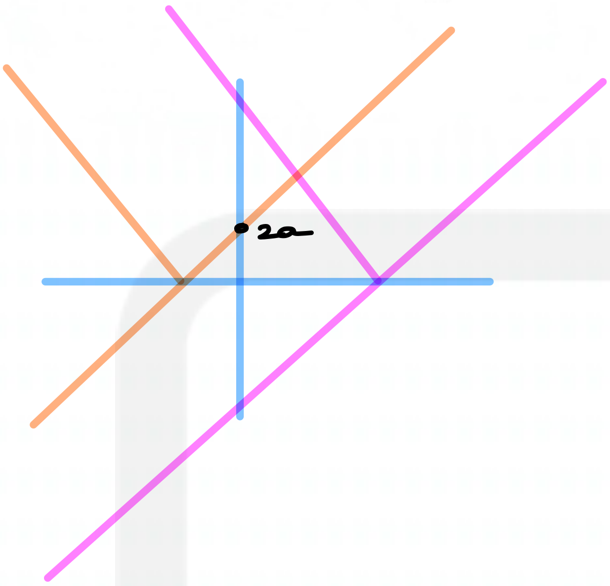
$= 3\ln 3 - 3\ln 2 + \ln 2 = 3\ln 3 - 2\ln 2$ M1 A1

Ans#8

8.	(a)	$\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
		$-kV$: k is constant of proportionality and the negative shows decrease (or loss)	
		giving $\frac{dV}{dt} = 20 - kV$ ✱ These Bs are to be awarded independently	B1
	(b)	$\int \frac{1}{20-kV}\,dV = \int 1\,dt$ separating variables	M1
		$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
		Using $V = 0, t = 0$ to evaluate the constant of integration	M1
		$c = -\frac{1}{k}\ln 20$	
		$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
		Obtaining answer in the form $V = A + Be^{-kt}$	M1
		$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}\left(1-e^{-kt}\right)$	A1
	(c)	$\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
		$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
		At $t = 10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1
	Alternative to (b)		
		Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
		Substituting into differential equation	
		$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
		$A = \frac{20}{k}$	M1 A1
		Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$	M1
		$B = -\frac{20}{k}$	A1

1. **(a)** Sketch, on the same diagram, the graphs of $y = |3x + 2a|$ and $y = |3x - 4a|$, where a is a positive constant.

Give the coordinates of the points where each graph meets the axes. [3]



(b) Find the coordinates of the point of intersection of the two graphs. [3]

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(c) Deduce the solution of the inequality $|3x + 2a| < |3x - 4a|$. [1]

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4. Given that 4 and $-3+2i$ are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation, $-3-2i$

(1)

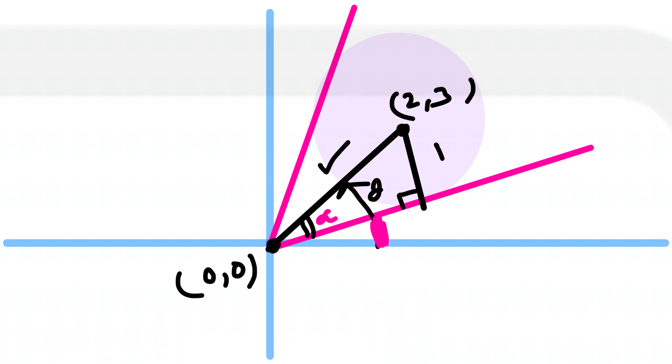
(b) find the value of a and the value of b.

(4)

$$(x-4)\,(x-(-3-2i))\,(x-(-3+2i)) = 0$$

(c) **(i)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z - 2 - 3\mathrm{i}| = 1$. [2]

$|z - (2+3i)| = 1$



(ii) Calculate the least value of arg z for points on this locus. [2]

8. Liquid is pouring into a container at a constant rate of $20\ \text{cm}^3\,\text{s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, $V\ \text{cm}^3$, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k.

(5)

Given also that $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(3)