Mock Exam 1

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

MATHEMATICS **9709**

Paper 3 Pure Mathematics 3 **1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3}\times$ base area $\times$ height

Area of curved surface of cone = $\pi r\times$ slant height

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2+bx+c=0$:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For an arithmetic series:

$$u_n = a+(n-1)d\,, \qquad S_n=\tfrac{1}{2}n(a+l)=\tfrac{1}{2}n\{2a+(n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}\,, \qquad S_n=\frac{a(1-r^n)}{1-r}\quad(r\neq 1)\,, \qquad S_\infty=\frac{a}{1-r}\quad(|r|<1)$$

Binomial series:

$$(a+b)^n = a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\binom{n}{3}a^{n-3}b^3+\ldots+b^n\text{, where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r}=\frac{n!}{r!(n-r)!}$$

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots\text{, where } n \text{ is rational and } |x|<1$$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \qquad 1+\tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
x^n	nx^{n-1}
$\ln x$	$\dfrac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
uv	$v\dfrac{\mathrm{d}u}{\mathrm{d}x} + u\dfrac{\mathrm{d}v}{\mathrm{d}x}$
$\dfrac{u}{v}$	$\dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

If $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \div \dfrac{\mathrm{d}x}{\mathrm{d}t}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$	
x^n	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x \rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$	$(\lvert x\rvert < a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln\lvert\mathrm{f}(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

1 Solve the inequality $2|x+2| > |3x-1|$. [4]

2 Find the coefficient of x^3 in the expansion of $(3-x)(1+3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

3 The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

4 Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

5 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\dfrac{dy}{dx} = \operatorname{cosec} 2t$. [5]

6 The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

7 (i) By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

(ii) Show by calculation that this root lies between $x = 1$ and $x = 1.5$. [2]

....

....

....

....

....

....

....

....

....

....

....

....

(iii) Use the iterative formula $x_{n+1} = \ln\left(\dfrac{4}{\ln(x_n + 2)}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N.$$ [1]

....

....

....

....

....

....

....

....

(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

....

....

....

....

....

....

....

....

....

....

....

....

(iii) State what happens to the value of N as t becomes large. [1]

Throughout this question the use of a calculator is not permitted.

9 The complex number u is defined by

$$u = \frac{4\text{i}}{1 - (\sqrt{3})\text{i}}.$$

(i) Express u in the form $x + \text{i}y$, where x and y are real and exact. [3]

(ii) Find the exact modulus and argument of u. [2]

....

....

....

....

....

....

....

....

....

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

10 With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

....

....

....

....

....

....

....

....

....

....

....

(b) Find a vector equation for the line through M and N. [2]

....

....

....

....

....

....

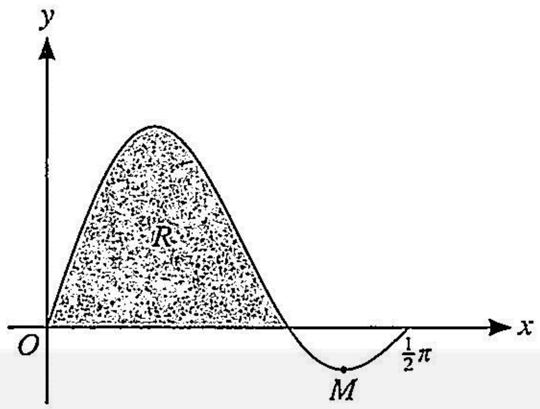
....

....

....

(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

11



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x). \qquad [3]$$

....

....

....

....

....

....

....

....

(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

....

....

....

....

....

....

(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.