Mock Exam 1

MATHEMATICS **9709**

Paper 3 Pure Mathematics 3

MARK SCHEME

Maximum Mark: 75

Published

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1 Solve the inequality $2|x+2| > |3x-1|$. [4]

State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x+2) = \pm(3x-1)$
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x
Obtain critical values $x = -\frac{3}{5}$ and $x = 5$
State final answer $-\frac{3}{5} < x < 5$

Alternative method for question 1

Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality
Obtain critical value $x = -\frac{3}{5}$ similarly
State final answer $-\frac{3}{5} < x < 5$

Find the coefficient of x^3 in the expansion of $(3 - x)(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\frac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$
State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\frac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$
Multiply by $(3 - x)$ to give 2 terms in x^3, or their coefficients
Obtain answer 6

The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

Substitute $x = -\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8} + \frac{1}{4}a - \frac{1}{2}b - 2 = 0$
Substitute $x = -2$ and equate result to -24
Obtain a correct equation, e.g. $-48 + 4a - 2b - 2 = -24$
Solve for a or for b
Obtain $a = 5$ and $b = -3$

Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$
Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent
Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent
Use limits correctly, having integrated twice
Obtain given answer correctly

The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\frac{dy}{dx} = \operatorname{cosec} 2t$. [5]

State $\frac{dx}{dt} = 2 + 2\cos 2t$
Use the chain rule to find the derivative of y
Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1 - \cos 2t}$
Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$
Obtain $\frac{dy}{dx} = \text{cosec}2t$ correctly

The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

State $4xy + 2x^2 \dfrac{dy}{dx}$, or equivalent, as derivative of $2x^2 y$
State $y^2 + 2xy \dfrac{dy}{dx}$, or equivalent, as derivative of xy^2
Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)
Reject $y = 0$
Obtain $y = 4x$
Obtain an equation in y (or in x) and solve for y (or for x) in terms of a
Obtain $y = -2a$

Alternative method for question 5
Rewrite as $y = \dfrac{a^3}{2x^2 - xy}$ and differentiate
Obtain correct derivative (in any form)
set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)
Obtain $4x - y = 0$
Confirm $2x^2 - xy \neq 0$
Obtain an equation in y (or in x) and solve for y (or for x)
Obtain $y = -2a$

(i) By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

(ii) Show by calculation that this root lies between $x = 1$ and $x = 1.5$. [2]

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(iii) Use the iterative formula $x_{n+1} = \ln\left(\frac{4}{\ln(x_n + 2)}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Sketch a relevant graph, e.g. $y = \ln(x+2)$
Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement
Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$
Complete the argument correctly with correct calculated values

Use the iterative formula correctly at least **twice** using output from a previous iteration
Obtain final answer 1.23
Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)

The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N.$$ [1]

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(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

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(iii) State what happens to the value of N as t becomes large. [1]

State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$
Separate variables correctly and integrate at least one side
Obtain term $\ln N$
Obtain term $0.5e^{-0.02t}$
Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a\ln N$ and $be^{-0.02t}$, where $ab \neq 0$
Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$
Substitute $N = 800$ and obtain $t = 29.6$
State that N approaches $\frac{1000}{\sqrt{e}}$

Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{4\text{i}}{1 - (\sqrt{3})\text{i}}.$$

(i) Express u in the form $x + \text{i}y$, where x and y are real and exact. [3]

(ii) Find the exact modulus and argument of u. [2]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

Multiply numerator and denominator by $1 + \sqrt{3}\mathrm{i}$, or equivalent
$4\mathrm{i} - 4\sqrt{3}$ and $3 + 1$
Obtain final answer $-\sqrt{3} + \mathrm{i}$

State that the modulus of u is 2
State that the argument of u is $\frac{5}{6}\pi$ (or 150°)
Show a circle with centre the origin and radius 2
Show u in a relatively correct position
Show the perpendicular bisector of the line joining u and the origin
Shade the correct region

With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

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(b) Find a vector equation for the line through M and N. [2]

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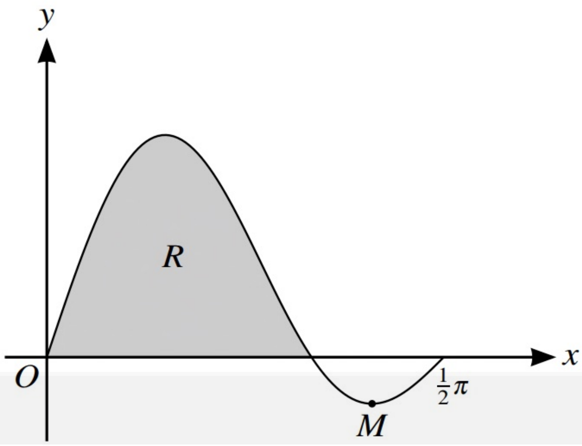
(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

State $\overrightarrow{OM} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$
Use a correct method to find $\overrightarrow{ON}$
Obtain answer $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$
Carry out a correct method to form a vector equation for MN
Obtain a correct equation in any form, e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$

State a correct vector equation for AB in any form, e.g. $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$
Equate components of AB and MN and solve for λ or for μ
Obtain $\lambda = -3$ or $\mu = 2$
Obtain position vector $\begin{pmatrix} -1 \\ 10 \\ 3 \end{pmatrix}$, or equivalent, for Q



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x).$$ [3]

(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

(iii) Using the result of part (i), express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$
Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$
Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly
Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$
Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly
Obtain answer $\frac{9}{16}$

State correct derivative $2\cos 4x+\cos 2x$
Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero
Obtain $4\cos^2 2x+\cos 2x-2=0$
Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$
Obtain answer $x=1.29$ only

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.