Cambridge Assessment International Education

# Cambridge International AS & A Level

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| CANDIDATE NAME | | | | | | | | | |
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*4961494717*

**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

**1** A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where $k$ is a constant.

Show that the line and the curve meet for all values of $k$. [4]

**2** A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the $x$-axis with scale factor $\frac{1}{2}$

Reflection in the $y$-axis

Stretch parallel to the $y$-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where $a$, $b$ and $c$ are constants. [4]

**3** A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the $x$-coordinate of the point on the curve at which the $x$- and $y$-coordinates are increasing at the same rate. [4]

**4** The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

**(a)** Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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**(b)** Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

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**5** Points $A$ (7, 12) and $B$ lie on a circle with centre (−2, 5). The line $AB$ has equation $y = -2x + 26$.

Find the coordinates of $B$. [6]

**6** In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant $a$. [6]

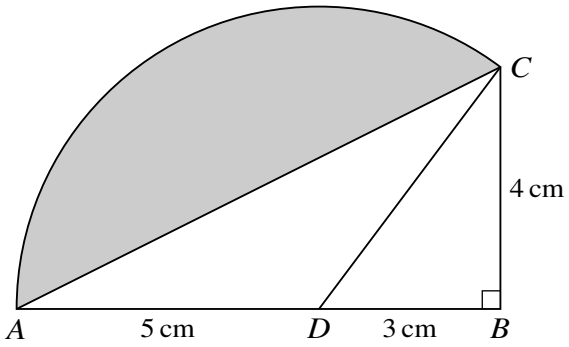
**7** **(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta \sin\theta = 1$$

for $0° < \theta < 360°$. [5]

**(b)** Show that $\frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

**8**




The diagram shows triangle $ABC$ in which angle $B$ is a right angle. The length of $AB$ is 8 cm and the length of $BC$ is 4 cm. The point $D$ on $AB$ is such that $AD = 5$ cm. The sector $DAC$ is part of a circle with centre $D$.

**(a)** Find the perimeter of the shaded region. [5]

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**(b)** Find the area of the shaded region. [3]

**9** The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

**(a)** State the range of f. [1]

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**(b)** Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

**(c)** Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

**10** At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-\frac{1}{2}} + k$, where $k$ is a constant.

**(a)** Show that $k = -2$. [1]

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**(b)** Find the equation of the curve. [4]

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**(c)** Find the coordinates of the stationary point. [3]

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**(d)** Determine the nature of the stationary point. [2]

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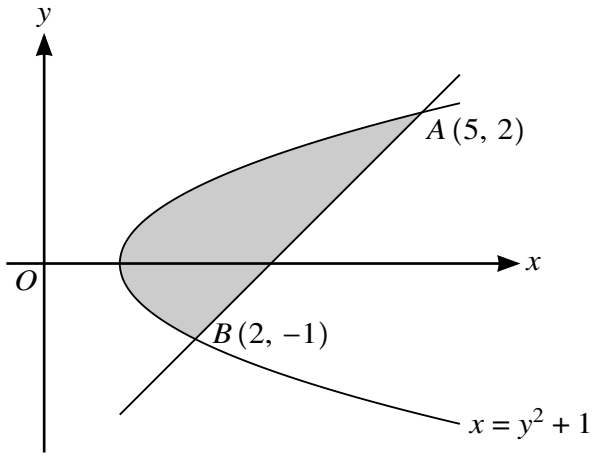
**11**




The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

**(a)** Find an equation of the line $AB$. [2]

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**(b)** Find the volume of revolution when the region between the curve and the line $AB$ is rotated through 360° about the ***y*-axis**. [9]

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## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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