Mock Exam 1

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709**

Paper 1 Pure Mathematics 1 **1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

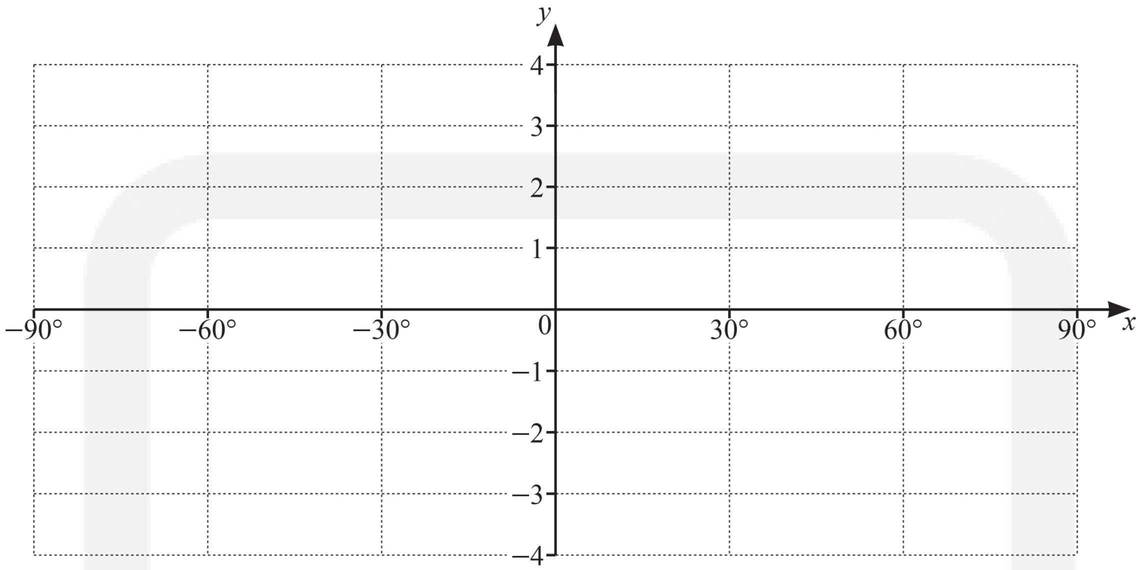
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 72.
- The number of marks for each question or part question is shown in brackets [ ].

1. On the axes below, sketch the graph of $y = 2\cos 3x - 1$ for $-90° \leqslant x \leqslant 90°$.



[3]

2.

(i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

3.

The functions f and g are defined by

$$\mathrm{f}(x) = 5x-2 \quad \text{for } x > 1,$$
$$\mathrm{g}(x) = 4x^2-9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $\mathrm{gf}(x) = 4$. [3]

4.

(a) Find the first 3 terms in the expansion of $\left(4-\frac{x}{16}\right)^6$ in ascending powers of x. Give each term in its simplest form. [3]

(b) Hence find the term independent of x in the expansion of $\left(4-\frac{x}{16}\right)^6\left(x-\frac{1}{x}\right)^2$. [3]

5.

A geometric progression has a second term of $27p^2$ and a fifth term of p^5. The common ratio, r, is such that $0 < r < 1$.

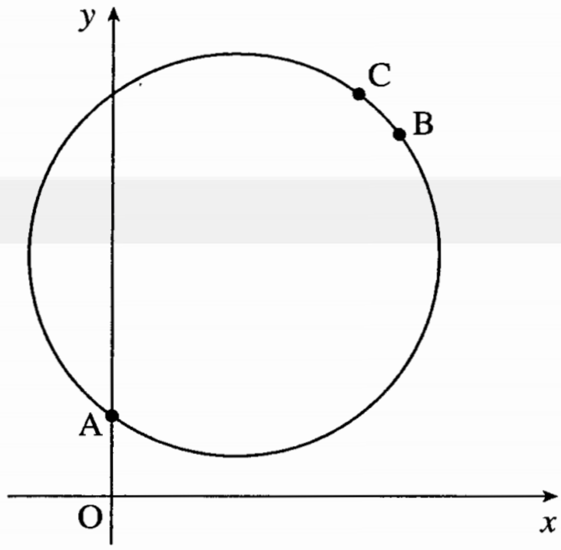
(i) Find r in terms of p. [2]

(ii) Hence find, in terms of p, the sum to infinity of the progression. [3]

(iii) Given that the sum to infinity is 81, find the value of p. [2]

6.

The points $A(0, 2)$, $B(7, 9)$ and $C(6, 10)$ lie on the circumference of a circle, as shown



(i) Find the length of AC.

Prove that triangle ABC is right-angled at B. [3]

(ii) Hence show that the centre of the circle is $(3, 6)$ and its radius is 5.

Find the equation of the circle. [3]

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7.

(a) Solve $6\sin^2 x - 13\cos x = 1$ for $0° \leqslant x \leqslant 360°$. [4]

(b) **(i)** Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\frac{4\tan y}{\sqrt{1+\tan^2 y}}$ can be written in the form $a\sin y$, where a is an integer. [3]

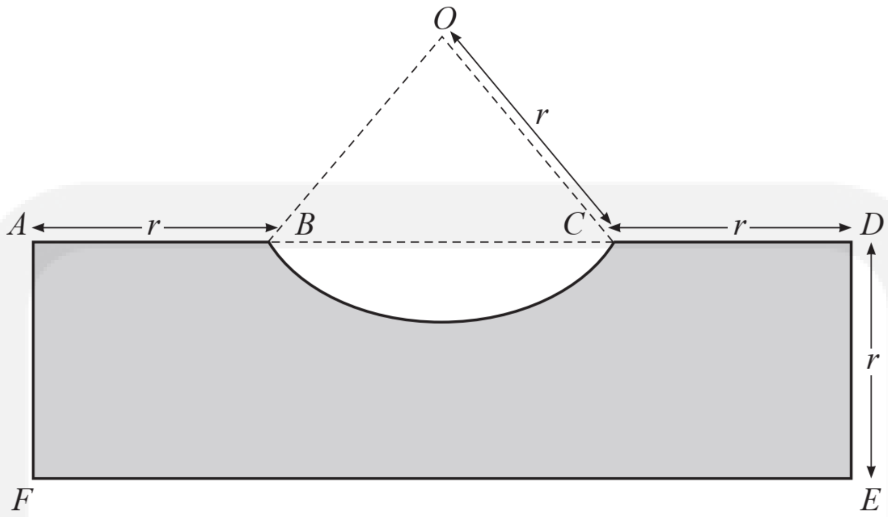
(ii) Hence solve $\frac{4\tan y}{\sqrt{1+\tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ radians. [1]

8.

A curve is such that $\dfrac{d^2y}{dx^2} = 2(3x-1)^{-\frac{2}{3}}$. Given that the curve has a gradient of 6 at the point (3, 11), find the equation of the curve. [8]

9.

In this question all lengths are in centimetres and all angles are in radians.

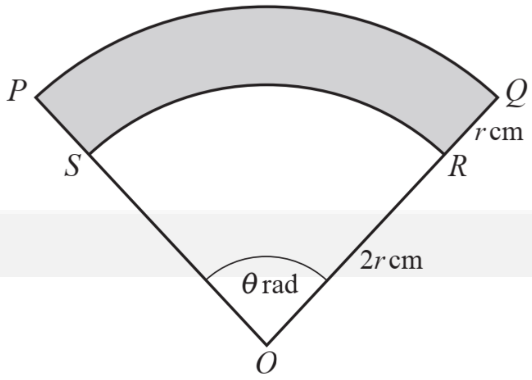


The diagram shows the rectangle $ADEF$, where $AF = DE = r$. The points B and C lie on AD such that $AB = CD = r$. The curve BC is an arc of the circle, centre O, radius r and has a length of $1.5r$.

(a) Show that the perimeter of the shaded region is $(7.5 + 2\sin 0.75)r$. [4]

(b) Find the area of the shaded region, giving your answer in the form kr^2, where k is a constant correct to 2 decimal places. [4]

10.



The diagram shows a sector OPQ of the circle centre O, radius $3r$ cm. The points S and R lie on OP and OQ respectively such that ORS is a sector of the circle centre O, radius $2r$ cm. The angle $POQ = \theta$ radians. The perimeter of the shaded region $PQRS$ is 100 cm.

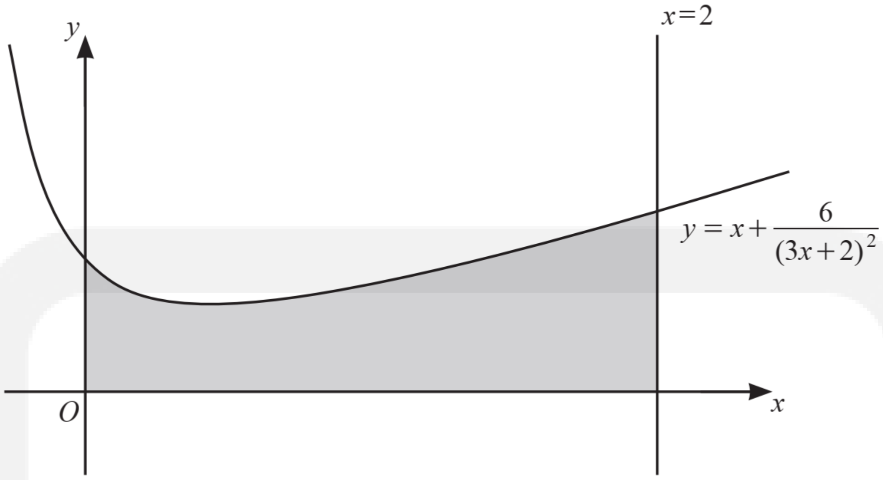
(i) Find θ in terms of r. [2]

(ii) Hence show that the area, A cm^2, of the shaded region $PQRS$ is given by $A = 50r - r^2$. [2]

(iii) Given that r can vary and that A has a maximum value, find this value of A. [2]

(iv) Given that A is increasing at the rate of $3\,\text{cm}^2\,\text{s}^{-1}$ when $r = 10$, find the corresponding rate of change of r. [3]

11.



The diagram shows part of the curve $y = x + \dfrac{6}{(3x+2)^2}$ and the line $x = 2$.

(i) Find, correct to 2 decimal places, the coordinates of the stationary point. [6]

(ii) Find the area of the shaded region, showing all your working. [4]

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