Mock Exam 1

MATHEMATICS **9709**

Paper 1 **1 hour 50 minutes**

MARK SCHEME

Maximum Mark: 72

Published

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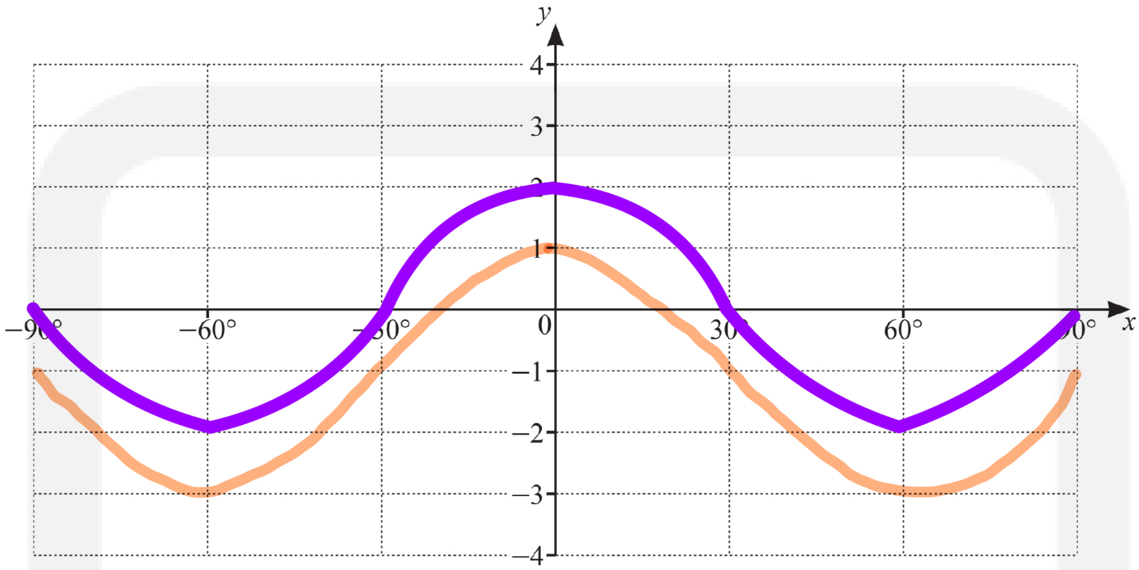
TRIG SKETCH

1. On the axes below, sketch the graph of $y = 2\cos 3x - 1$ for $\overbrace{-90° \leqslant x \leqslant 90°}^{180°}$.

3 cycles in 360°

1½ ← 180°



[3]

2. QUADRATICS

(i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

$5[x^2 - 3x + \frac{1}{5}]$

$5[x^2 - 3x + (-\frac{3}{2})^2 - (-\frac{3}{2})^2 + \frac{1}{5}]$

$5[(x - \frac{3}{2})^2 - \frac{9}{4} + \frac{1}{5}]$

$5[(x - \frac{3}{2})^2 - \frac{41}{20}]$

$5(x - \frac{3}{2})^2 - \frac{41}{4}$

OR

$5(x - 1.5)^2 - 10.25$

(ii) Hence state the least value of $\underbrace{x^2 - 3x + 0.2}$ and the value of x at which this occurs. [2]

$[x^2 - 3x + \frac{1}{5}]$

$(x - \frac{3}{2})^2 - 2.05$

$a(x - h)^2 + k$

MIN VALUE

$h = \frac{3}{2}$, $k = -2.05$

value of x Least value

3. **FUNCTIONS**

The functions f and g are defined by

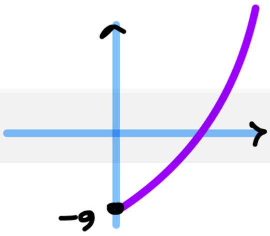
$$f(x) = 5x - 2 \quad \text{for } x > 1,$$
$$g(x) = 4x^2 - 9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

$g(x) = 4x^2 - 9$

$D: x > 0$

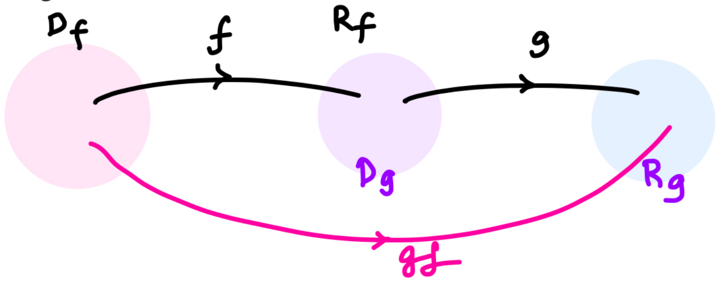
$R: g(x) > -9$



$\Rightarrow D_f : x > 1$

(ii) Find the domain of gf. [1]

gf means first f then g



(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

$$gf(x) = 4(f(x))^2 - 9$$
$$= 4(5x-2)^2 - 9$$

$$gf(x) = 4$$

$$4(5x-2)^2 - 9 = 4$$

$$4(5x-2)^2 = 13$$

$$(5x-2)^2 = \frac{13}{4}$$

$$(5x-2) = \pm\frac{\sqrt{13}}{2}$$

$$5x = 2 \pm \frac{\sqrt{13}}{2}$$

$$x = \frac{1}{5}\left(2 \pm \frac{\sqrt{13}}{2}\right)$$

4. **BINOMIAL EXP.**

(a) Find the first 3 terms in the expansion of $\left(4-\frac{x}{16}\right)^6$ in ascending powers of x. Give each term in its simplest form. [3]

$$\left(4-\frac{x}{16}\right)^6 = {}^6C_0(4)^6\left(-\frac{x}{16}\right)^0 + {}^6C_{\boxed{1}}(4)^5\left(-\frac{x}{16}\right)^1 + {}^6C_2(4)^4\left(-\frac{x}{16}\right)^2 + \cdots$$

$$= 4^6 + 6(4)^5\left(-\frac{x}{16}\right) + 15(4)^4\left(\frac{x^2}{256}\right) + \cdots$$

$$= 4096 - 384x + 15x^2$$

$$\left(x-\frac{1}{x}\right)^2 = x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$$

$$= x^2 - 2 + \frac{1}{x^2}$$

(b) Hence find the term independent of x in the expansion of $\left(4-\frac{x}{16}\right)^6\left(x-\frac{1}{x}\right)^2$. [3]

$$\left(\boxed{4096} - 384x + \boxed{15}x^2\right)\left(x^2 \boxed{-2} + \frac{\boxed{1}}{x^2}\right)$$

indep of x ⟶ const. Term

$$4096(-2) + 15(1)$$

$$-8177$$

5. GP

A geometric progression has a second term of $27p^2$ and a fifth term of p^5. The common ratio, r, is such that $0 < r < 1$.

$T_2 = 27p^2$
$T_5 = p^5$

(i) Find r in terms of p. [2]

$ar^4 = p^5$ —— $\boxed{1}$
$ar = 27p^2$ —— $\boxed{2}$

$\frac{ar^4}{ar} = \frac{p^5}{27p^2}$

$r^3 = \frac{p^3}{27}$

$r = \frac{p}{3}$

(ii) Hence find, in terms of p, the sum to infinity of the progression. [3]

$T_2 = 27p^2$
$ar = 27p^2$
$a\left(\frac{p}{3}\right) = 27p^2$
$a = 81p$

$S_\infty = \frac{a}{1-r}$
$= \frac{81p}{1-\frac{p}{3}}$
$= (81p) \div \left(\frac{3-p}{3}\right)$
$= 81p\left(\frac{3}{3-p}\right)$
$= \frac{243p}{3-p}$

(iii) Given that the sum to infinity is 81, find the value of p. [2]

$\frac{243p}{3-p} = 81$

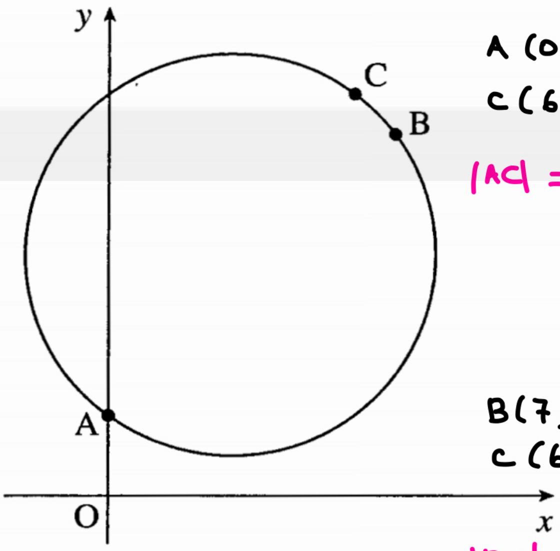
$243p = 81(3-p)$
$= 243 - 81p$
$324p = 243$
$p = 0.75$ or $3/4$

6. CIRCLE EQN

The points A(0, 2), B(7, 9) and C(6, 10) lie on the circumference of a circle, as shown

A (0,2)
B (7,9)

$|AB| = \sqrt{7^2+7^2}$
$= \sqrt{98}$ units



A (0,2)
C (6,10)

$|AC| = \sqrt{6^2+8^2}$
$= \sqrt{100}$ units

B(7,9)
C (6,10)

$|BC| = \sqrt{1^2+1^2}$
$= \sqrt{2}$ units

(i) Find the length of AC. ✓

Prove that triangle ABC is right-angled at B. [3]

means AC is the hyp.

$AC^2 = AB^2 + BC^2$ → pythagora's Theorem

$(\sqrt{100})^2 = (\sqrt{98})^2 + (\sqrt{2})^2$

$100 = 98 + 2$ ✓

(ii) Hence show that the centre of the circle is $(3, 6)$ and its radius is 5.

Find the equation of the circle.

[3]

Circle property → Diameter subtends $90°$ at circumf

Since $\hat{B} = 90°$

$\Rightarrow$ hyp → AC

AC → diameter

Dia $= |AC| = \sqrt{100}$

$= 10$ units

$\therefore$ $r = 5$ units

midpoint of A and C → centre of circle

A $(0, 2)$

C $(6, 10)$

$M\left(\frac{0+6}{2}, \frac{2+10}{2}\right)$

$M(3, 6)$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-3)^2 + (y-6)^2 = 25$

7. TRIG EQNS.

(a) Solve $6\sin^2 x - 13\cos x = 1$ for $0° \leqslant x \leqslant 360°$. [4]

$6(1-\cos^2 x) - 13\cos x - 1 = 0$

$6 - 6\cos^2 x - 13\cos x - 1 = 0$

$-6\cos^2 x - 13\cos x + 5 = 0$

$6\cos^2 x + 13\cos x - 5 = 0$

$6\cos^2 x - 2\cos x + 15\cos x - 5 = 0$

$2\cos x(3\cos x - 1) + 5(3\cos x - 1) = 0$

$\cos x = 1/3$ $\quad\quad$ $\cos x = -5/2$

No Soln

Quad : I/IV

Range : $0° \le x \le 360°$

Basic ∡ : $\cos\alpha = 1/3$

$\alpha = 70.5°$

Final Ans : $x = 70.5°, 360° - 70.5°$

$= 70.5°, 289.5°$

(b) (i) Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\dfrac{4\tan y}{\sqrt{1+\tan^2 y}}$ can be written in the form $a \sin y$, where a is an integer. [3]

N: $4\tan y$

$4\dfrac{\sin y}{\cos y}$

D: $\sqrt{1+\tan^2 y}$

: $\sqrt{\left(1+\dfrac{\sin^2 y}{\cos^2 y}\right)}$

: $\sqrt{\dfrac{\cos^2 y+\sin^2 y}{\cos^2 y}}$

: $\sqrt{\dfrac{1}{\cos^2 y}}$

: $\dfrac{1}{\cos y}$

$\dfrac{N}{D}$ $\left(\dfrac{4\sin y}{\cos y}\right) \div \left(\dfrac{1}{\cos y}\right)$

$\dfrac{4\sin y}{\cos y} \cdot \dfrac{\cos y}{1}$

$4\sin y$

$a = 4$

(ii) Hence solve $\dfrac{4\tan y}{\sqrt{1+\tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ radians. [1]

$4\sin y + 3 = 0$

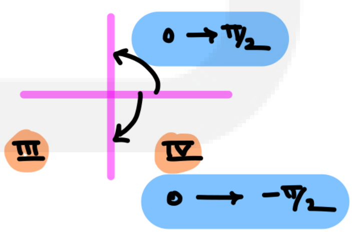
$\sin y = -3/4$

Quad : III / IV

Range : $-\frac{\pi}{2} < y < +\pi/2$

Basic ∠ : $\sin y = +0.75$

$y = 0.8481$



ONLY applicable Quad is 4th Quad

⇒ Final Ans

$y = -0.848$ (3 sf.)

8. INTEGRATION

A curve is such that $\frac{d^2y}{dx^2} = 2(3x-1)^{-\frac{2}{3}}$. Given that the curve has a gradient of 6 at the point (3, 11), find the equation of the curve. [8]

$$\frac{dy}{dx} = \int 2(3x-1)^{-2/3}\,dx$$

$$= \frac{2[3x-1]^{-2/3+1}}{(3)(-2/3+1)} + c$$

$$\frac{dy}{dx} = 2(3x-1)^{1/3} + c$$

$x = 3$, $\frac{dy}{dx} = 6$

$$6 = 2[8]^{1/3} + c$$

$$6 = 2[2] + c$$

$$\therefore c = 2$$

$$y = \int \left(2(3x-1)^{1/3} + 2\right) dx$$

$$= \frac{2[3x-1]^{1/3+1}}{(3)(1/3+1)} + 2x + K$$

$$y = \frac{1}{2}[3x-1]^{4/3} + 2x + K$$

$x = 3$, $y = 11$

$$11 = \frac{1}{2}[8]^{4/3} + 2(3) + K$$

$$11 = 8 + 6 + K$$

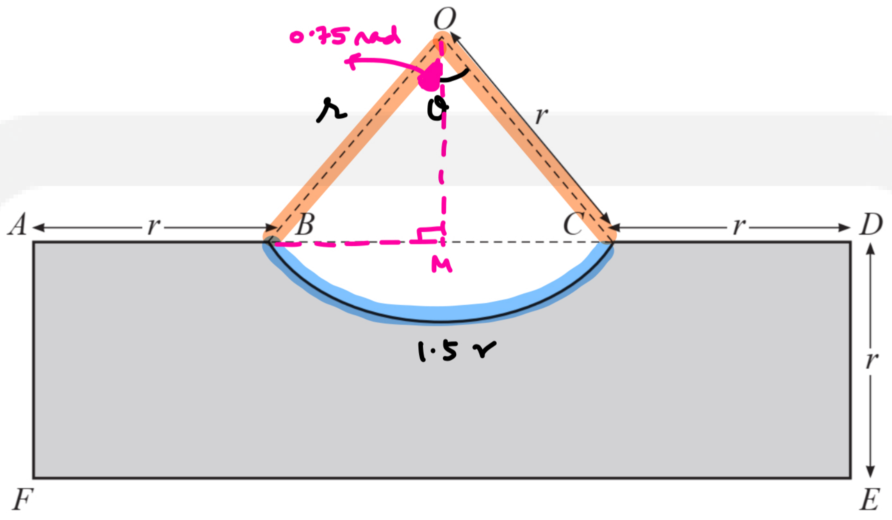
$$\underline{K = -3}$$

Equ. of curve

$$y = \frac{1}{2}(3x-1)^{4/3} + 2x - 3$$

9. **CM**

In this question all lengths are in centimetres and all angles are in radians.



The diagram shows the rectangle $ADEF$, where $AF = DE = r$. The points B and C lie on AD such that $AB = CD = r$. The curve BC is an arc of the circle, centre O, radius r and has a length of $1.5r$.

(a) Show that the perimeter of the shaded region is $(7.5 + 2\sin 0.75)r$. [4]

$s = r\theta$

$1.5r = r\theta$

$\theta = 1.5$

$\sin 0.75 = \dfrac{BM}{r}$

$BM = r\sin 0.75$

$BM = MC$

$BC = 2BM$

$= 2r\sin 0.75$

$FE = AB + BC + CD$

$= r + 2r\sin 0.75 + r$

$= 2r + 2r\sin 0.75$

$P_{\text{shaded}} = FE + \overset{r}{AF} + \overset{r}{DE} + \overset{r}{AB} + \overset{r}{CD} + \overset{1.5r}{\text{arc } BC}$

$= 2r + 2r\sin 0.75 + 4(r) + 1.5r$

$= 7.5r + 2r\sin 0.75$

$= (7.5 + 2\sin 0.75)r \longrightarrow$ QED! ✓

(b) Find the area of the shaded region, giving your answer in the form kr^2, where k is a constant correct to 2 decimal places. [4]

Area of Δ OBC $= \frac{1}{2} \cdot r \cdot r \cdot \sin(1.5)$

— " — of sector OBC $= \frac{1}{2} r^2 (1.5)$

$\therefore$ Area of segment BC $= \frac{1}{2} r^2 [1.5 - \sin 1.5]$

$= \frac{1}{2} r^2 [0.5025]$

Area of Rectangle $= (AF)(FE)$

$= (r)(2r + 2r \sin 0.75)$

$= r[2r(1 + \sin 0.75)]$

$= 2r^2 [1 + \sin 0.75]$

$= 2r^2 [1.682]$

Area of shaded region $= A_{\square} - A_{\frown}$

$= 2r^2 [1.682] - \frac{1}{2} r^2 [0.5025]$

$= r^2 \left[2[1.682] - \frac{1}{2}[0.5025] \right]$

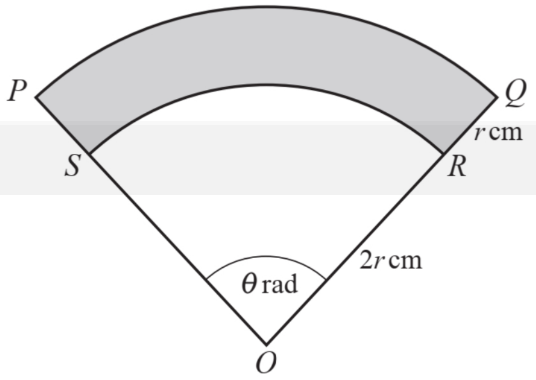
$= r^2 (3.364 - 0.25125)$

$= r^2 (3.11275)$

FINAL ANS

$3.11 r^2$ $\quad$ $k = 3.11$ (2 dp)

10. Diff^n → st. pts
→ Rates



The diagram shows a sector OPQ of the circle centre O, radius $3r$ cm. The points S and R lie on OP and OQ respectively such that ORS is a sector of the circle centre O, radius $2r$ cm. The angle $POQ = \theta$ radians. The perimeter of the shaded region $PQRS$ is 100 cm.

(i) Find θ in terms of r. [2]

$S_{RS} = (2r)\theta$

$S_{QP} = (3r)\theta$

$P_{\text{shaded}} = r + r + 2r\theta + 3r\theta$
$= 2r + 5r\theta$

$2r + 5r\theta = 100$

$5r\theta = 100 - 2r$

$\theta = \dfrac{100 - 2r}{5r}$

$= \dfrac{20}{r} - \dfrac{2}{5}$

(ii) Hence show that the area, A cm^2, of the shaded region $PQRS$ is given by $A = 50r - r^2$. [2]

$A_{ORS} = \frac{1}{2}(2r)^2\theta$
$= \frac{1}{2}(4r^2)\theta$
$= 2r^2\theta$

$A_{OQP} = \frac{1}{2}(3r)^2\theta$
$= \frac{1}{2}(9r^2)\theta$
$= 4\frac{1}{2}r^2\theta$

$A_{\text{shaded}} = A_{OQP} - A_{ORS}$
$= 2\frac{1}{2}r^2\theta$
$= \frac{5}{2}r^2\theta$

But $\theta = \dfrac{100 - 2r}{5r}$

Area $= \dfrac{5}{2}\boxed{r^2}\left[\dfrac{100 - 2r}{5\boxed{r}}\right]$

$= \dfrac{r}{2}\left[100 - 2r\right]$

$= 50r - r^2$ QED!

(iii) Given that r can vary and that A has a maximum value, find this value of A. [2]

$A = 50r - r^2$

$A' = 50 - 2r$

$A' = 0$

$\therefore 50 - 2r = 0$

$r = 25$ cm

$A = 50(25) - (25)^2$

$= 25[50 - 25]$

$= 625 \text{ cm}^2$

(iv) Given that A is increasing at the rate of $3\,\text{cm}^2\,\text{s}^{-1}$ when $r = 10$, find the corresponding rate of change of r. [3]

$\frac{dA}{dt} = 3 \frac{\text{cm}^2}{\text{s}}$

$r = 10, \quad \frac{dA}{dr} = 50 - 2(10)$

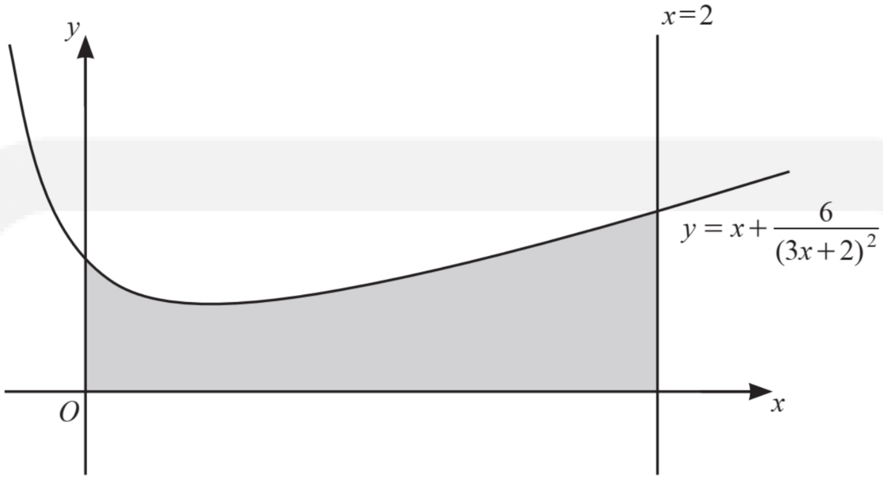
$= 30$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$3 = 30 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{10}$ OR $0.1 \frac{\text{cm}}{\text{s}}$

11. ST-POINTS + AREA under graph



The diagram shows part of the curve $y = x + \frac{6}{(3x+2)^2}$ and the line $x = 2$.

(i) Find, correct to 2 decimal places, the coordinates of the stationary point.

$y = x + 6(3x+2)^{-2}$

$\frac{dy}{dx} = 1 + 6(-2)(3x+2)^{-3}(3)$

$= 1 - \frac{36}{(3x+2)^3}$

$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{36}{(3x+2)^3} = 0$

$\frac{36}{(3x+2)^3} = 1$

$(3x+2)^3 = 36$

$(3x+2) = \sqrt[3]{36}$

$x = \frac{(\sqrt[3]{36}) - 2}{3}$

$x = 0.4339$

$x = 0.43$ (2 dp)

$y = 0.98$ (2 dp)

(ii) Find the area of the shaded region, showing all your working. [4]

$$A = \int \left(x + \frac{6}{(3x+2)^2} \right) dx$$

$$= \int \left(x + 6(3x+2)^{-2} \right) dx$$

$$= \frac{x^2}{2} + 6 \left[\frac{(3x+2)^{-2+1}}{(3)(-2+1)} \right]$$

$$= \left[\frac{x^2}{2} - \frac{2}{(3x+2)} \right]_0^2$$

2.75 square units

The End!

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